

ENTRANCE TEST FOR Ph.D. PROGRAMME, 2023

STATISTICS

Time : Three Hours

Maximum : 100 Marks

Part A

*Answer all questions.**Each question carries 1 mark.*

Choose the correct answer from the choices :

- The problem is to infer about the parameter $p \in (0, 1)$ of a discrete distribution. Beta distribution of first kind is the conjugate prior when the parent distribution is _____.
(A) Binomial. (B) Negative binomial.
(C) Geometric. (D) All of these.
- Which of the following tests is not based on empirical DF ?
(A) Anderson-Darling test. (B) χ^2 - goodness of fit test.
(C) Kolmogorov-Smirnov test. (D) Watson test.
- _____ technique/method allows estimation of the sampling distribution of almost any statistic using random sampling methods.
(A) Bootstrap. (B) EDA.
(C) Delta. (D) Classical.
- In SRSWOR for 10 units from a population of 100 units, the probability that a specified pair of distinct units will be included in the sample is _____.
(A) $\frac{1}{100}$. (B) $\frac{1}{110}$.
(C) $\frac{1}{90}$. (D) $\frac{9}{10}$.
- If the distribution function under H_0 is P_0 under H_1 is P_1 and they are equal, then the MP level α test with power β satisfies :
(A) $\alpha \neq \beta$. (B) $\alpha > \beta$.
(C) $\beta > \alpha$. (D) $\alpha = \beta$.

Turn over

6. Let X and Y be two independent random variables such that XY is degenerate at $c \neq 0$. Which of the following statement is true ?
- (A) Only X is degenerate. (B) Only Y is degenerate.
 (C) Both X and Y are degenerate. (D) Neither X nor Y is degenerate.
7. With respect to a 2^2 factorial experiment involving two factors A and B , the factor A has significant main effect means that the mean response at :
- (A) The two levels of A are significantly different.
 (B) Different levels of A differ significantly at different levels of B .
 (C) All level combinations are significantly different.
 (D) The two levels of B may not be significantly different.
8. In a χ^2 test of independence between sex and kinds of phobias, the null hypothesis was rejected. The proper conclusion is that :
- (A) Sex and phobias are independent of each other.
 (B) Sex and phobias are related to each other.
 (C) Knowing a person's phobia gives no clue to his/her sex.
 (D) None of the above.
9. If the joint density $f(X_1, X_2, \dots, X_n; \theta)$ of n random variables, X_1, X_2, \dots, X_n is considered to be a function of θ . Then $L(\theta; X_1, X_2, \dots, X_n)$ is called :
- (A) Maximum Likelihood function. (B) Likelihood function.
 (C) Log Likelihood Function. (D) Marginal Likelihood Function.
10. What is an appropriate graphical tool to know the properties of data measured on interval scale ?
- (A) Histogram. (B) Bar chart.
 (C) Pie chart. (D) None of (A), (B), (C).
11. Let T be an unbiased estimator of θ and $I(\theta) = E \left[\frac{\partial \log L(\theta)}{\partial \theta} \right]^2$. Then Cramer-Rao lower bound for $V(T)$ is :
- (A) $I(\theta)$. (B) $\sqrt{I(\theta)}$.
 (C) $\frac{1}{\sqrt{I(\theta)}}$. (D) $\frac{1}{I(\theta)}$.

12. Let X_1, X_2 be a random sample from $U(0, \theta), \theta > 0$. MLE of θ is _____.
- (A) $X_1 + X_2$. (B) $X_1 X_2$.
 (C) $\text{Max} \{X_1, X_2\}$. (D) $\text{Min} (X_1 X_2)$.
13. A test function takes values in _____.
- (A) $(0, \infty)$. (B) $(0, 1)$.
 (C) $(0, 1]$. (D) $[0, 1]$.
14. Let X follow $N(0, \sigma^2), \sigma > 0$. The family of distribution of X has MLR property in _____.
- (A) X . (B) X^2 .
 (C) $-X$. (D) $-X^2$.
15. Let $\{N(t), t \geq 0\}$ be a Poisson process with rate λ . Then conditional distribution of $N(t)$ given $N(s) = m$, for $s < t$ is :
- (A) Poisson. (B) Truncate Poisson.
 (C) Uniform. (D) Binomial.
16. Let $U \sim U(0, 1)$ and $0 < p < 1$ then integer part of $\left(\frac{\log_e(U)}{\log_e(1-p)}\right)$ is :
- (A) Binomial variate. (B) Poisson variate.
 (C) Geometric variate. (D) Hypergeometric variate.
17. Let X and Y be two independent Poisson r.v.s with parameters λ and θ respectively. Then which of the following statements is not correct ?
- (A) $P[X + Y = 5] = e^{-(\lambda+\theta)}(\lambda + \theta)^5 / 5!$.
 (B) $P[X \leq 5 | Y \leq 20] = \sum_{i=1}^5 e^{-\lambda} \lambda^i / i!$.
 (C) $P[X = 5 | X + Y = 10] = \binom{10}{5} (\lambda\theta)^5 / (\lambda + \theta)^{10}$.
 (D) $P[X - Y = 5] = e^{-(\lambda-\theta)}(\lambda - \theta)^5 / 5!$.

Turn over

18. Let X be a normal random variable with mean 1 and variance 1. Define events $E = \{-1 < X < 0\}$, $F = \{2 < X < 3\}$ and $G = \{0 < X < 2\}$. Then :
- (A) $P[E] = P[F] = P[G]$. (B) $P[E] = P[F] < P[G]$.
 (C) $P[E] = P[G] < P[F]$. (D) $P[F] = P[G] = P[E]$.
19. Let X_1, X_2, X_3 be a random sample from $U(0, \theta)$. Let $\hat{\theta}_M$ represent moment estimator and $\hat{\theta}_L$ represent the MLE of θ . Then which of the following is true ?
- (A) $\hat{\theta}_M = \hat{\theta}_L$. (B) $E(\hat{\theta}_M) = E(\hat{\theta}_L)$.
 (C) $MSE(\hat{\theta}_M) > MSE(\hat{\theta}_L)$. (D) $MSE(\hat{\theta}_M) < MSE(\hat{\theta}_L)$.
20. In how many ways 4 boys and 3 girls can be seated in a row so that they are alternate :
- (A) 144. (B) 288.
 (C) 12. (D) 256.
21. The Jeffreys prior is proportional to the :
- (A) Underlying pdf. (B) Underlying cdf.
 (C) Information matrix. (D) Square root of the information matrix.
22. Let X_1, X_2, \dots, X_n be an observed sample. Let $\{S_j, j = 1, 2, \dots, m\}$ be a prespecified partition of the sample space and N_j be the number of observations falling in S_j . Let P_j be the probability of S_j under a specified distribution, then the distribution of $Y = \sum_{j=1}^m \frac{(N_j - np_j)^2}{np_j}$ is :
- (A) χ_{m-1}^2 .
 (B) χ_{n-1}^2 .
 (C) Approaches χ_{m-1}^2 , as n increases.
 (D) Approaches χ_{n-1}^2 , as n increases.
23. The power of the MP test of size α for testing $H_0 : \theta = 1$ against $H_1 : \theta = 0$ based on a single observation from the distribution with pdf $f(x, \theta) = 2(x\theta + 1 - \theta)$, $0 < x < 1$, is :
- (A) $\sqrt{\alpha}$. (B) α .
 (C) $\frac{\alpha}{2}$. (D) 2α .

24. Size of the test is :
- (A) Always greater than or equal to the level of significance.
 - (B) Always less than or equal to the level of significance.
 - (C) Always equal to the level of significance.
 - (D) Some times greater than the level of significance.
25. Let $(X, Y) \sim$ Bivariate normal $(0, 0, \sigma_1^2, \sigma_2^2, \rho)$. Which of the following statements is wrong ?
- (A) X and Y are independent only if $\rho = 0$.
 - (B) $X + Y$ and $X - Y$ are independent only if $\rho = 0$.
 - (C) $X + Y$ and $X - Y$ are independent only if $\sigma_1^2 = \sigma_2^2$.
 - (D) $(X + Y, X - Y)$ is distributed as bivariate normal.
26. In a general linear model $y = X\beta + \varepsilon$, _____.
- (A) Any estimable linear parametric function is a linear combination of the functions $X\beta$.
 - (B) If S^- is a g -inverse of $S = X'X$ then S^-y is a least square estimator of β .
 - (C) The coefficient vector of any function belonging to the error space is orthogonal to the rows of X .
 - (D) A least square estimator of β is unbiased for β .
27. Which of the following distribution does not belong to one-parameter Cramer family of distributions ?
- (A) Double exponential distribution with location θ and scale 1.
 - (B) Double exponential distribution with location 1 and scale θ .
 - (C) Exponential distribution with rate θ and location 1.
 - (D) Poisson distribution.
28. Which of the following is true ?
- (A) Unbiased estimator is always consistent.
 - (B) Consistent estimator is always unbiased.
 - (C) Consistent estimator is unique.
 - (D) Maximum Likelihood Estimator (MLE) need not be unbiased.
29. If A and B be two events, then which of the following is true ?
- (A) $P(A \cap B) = P(A)P(B)$.
 - (B) $P(A \cap B) \geq 1 - P(A^c) - P(B^c)$.
 - (C) $P(A \cup B) > P(A) + P(B)$.
 - (D) $P(A \cap B) \leq P(A) + P(B) - 1$.

Turn over

30. Let A and B be two independent events. If $P(A) = 1/4$ and $P(B) = 1/3$, then $P(A^c/B^c)$ is :
- (A) $1/4$. (B) $9/8$.
(C) $1/12$. (D) $3/4$.
31. Suppose you have a coin with probability $1/4$ of getting a head. If you toss the coin twice independently, then what is the probability of getting at least one head ?
- (A) $1/4$. (B) $7/16$.
(C) $3/16$. (D) $3/8$.
32. A biased coin is tossed until a head appears for the first time. Let p be the probability of a head, $0 < p < 1$. The probability that the number of tosses required is odd is :
- (A) $\left(\frac{1}{2}\right)^p$. (B) $p/2$.
(C) $1/(2-p)$. (D) p .
33. If X follows Binomial (n, p) then $n - X$ follows :
- (A) Binomial (n, p) . (B) Binomial $(n, 1 - p)$.
(C) Binomial $(2n, p)$. (D) Binomial $(2n, 1 - p)$.
34. Let X be a non-negative random variable with distribution function F. Then $E(X)$ is :
- (A) $\int_0^{\infty} xF(x) dx$. (B) $\int_0^{\infty} F(x) dx$.
(C) $\int_0^{\infty} x[1 - F(x)] dx$. (D) $\int_0^{\infty} [1 - F(x)] dx$.
35. Let X be a random variable. Then which of the following is not always a random variable ?
- (A) $|X|$. (B) X^2 .
(C) $X^{1/2}$. (D) $|X|^{1/2}$.
36. Let X and Y be two random variables. Which of the following is true ?
- (A) $E[V(Y|X)] = V[E(Y|X)] + V(Y)$.
(B) $E[V(Y|X)] = V(Y|X)$.
(C) $E[V(Y|X)] = V(Y) - V[E(Y|X)]$.
(D) $E[V(Y|X)] = V(Y)$.
37. Mode of the Chi-square distribution with n d.f. is :
- (A) $n - 2$. (B) $2n$.
(C) n . (D) $n/2$.

38. Neyman-Pearson lemma is used to find a most powerful test for testing :
- (A) Simple hypothesis against composite alternative.
 - (B) Composite hypothesis against composite alternative.
 - (C) Composite hypothesis against simple alternative.
 - (D) Simple hypothesis against simple alternative.
39. Rejecting a null hypothesis when it is true ?
- (A) Type I error.
 - (B) Type II error.
 - (C) No error.
 - (D) Simple error.
40. The statistic H under the null hypothesis for Kruskal-Wallis test is approximately distributed :
- (A) Chi square distribution.
 - (B) t -distribution.
 - (C) F-distribution.
 - (D) Normal distribution.
41. The distribution used in sign test is :
- (A) Poisson distribution.
 - (B) Uniform distribution.
 - (C) Binomial distribution.
 - (D) Normal distribution.
42. The Bayes estimate of a parameter θ under absolute error loss function is :
- (A) Mean of the posterior distribution.
 - (B) Median of the posterior distribution.
 - (C) Mode of the posterior distribution.
 - (D) None of these.
43. The multiple correlation coefficient lies between :
- (A) 0 and ∞ .
 - (B) -1 and 1.
 - (C) 0 and 1.
 - (D) -1 and 0.
44. Wishart distribution is the multivariate analog of :
- (A) Normal distribution.
 - (B) F Distribution.
 - (C) t -distribution.
 - (D) Chi-square distribution.
45. Distribution of Hotelling's T^2 statistics is :
- (A) F distribution.
 - (B) Whishart distribution.
 - (C) Chi-square distribution.
 - (D) t distribution.

Turn over

46. In a Markov chain a recurrent state i is said to be null recurrent if and only if the mean recurrent time μ_i is :
- (A) Zero. (B) One.
(C) Infinity. (D) Negative.
47. In time series analysis simple average method is used to calculate :
- (A) Trend values. (B) Seasonal indices.
(C) Cyclic variations. (D) All of these.
48. Laspeyre's index formula uses the weights of the :
- (A) Current year.
(B) Base year.
(C) Average of the weights over a number of years.
(D) None of these.
49. Which of the following designs does not apply all the three basic principle of design of experiments ?
- (A) RBD. (B) CRD.
(C) LSD. (D) GLSD.
50. Error degrees of freedom for a Graeco Latin square design of size 5 is :
- (A) 8. (B) 24.
(C) 4. (D) 12.

(50 × 1 = 50 marks)

Part B

*Answer any ten questions.
Each question carries 5 marks.*

51. Explain one-way and two-way ANOVA.
52. Differentiate between Correlation and Regression analysis.
53. What is Chi-square test ? Explain its significance in statistical analysis for any research problem.
54. Write a note on positive and negative correlation.
55. What is the relevance of questionnaire in data collection and interpretation ?
56. What is hypothesis testing ? Explain.
57. Explain in detail about the different steps involved in developing a research plan.
58. What is systematic sampling ? Explain.
59. Describe sampling and non-sampling errors.

60. What are the characteristics of "Completely Randomized Design" ?
61. What is the principle behind sample size calculation ? Explain the factors while determining the sample size.
62. Explain the features of Latin Square Design.
63. What do you mean by UMP test ? Explain.
64. Explain the methods of graphical representation of data.

(10 × 5 = 50 marks)